

# Analysis and Synthesis of Triplate Branch-Line 3dB Coupler based on the Planar Circuit Theory

Tetsuo Anada and Hsu, Jui-Pang

Faculty of Engineering, KANAGAWA UNIVERSITY  
221 Yokohama-shi JAPAN

## Abstract

This paper is presented how to analyze and synthesize a triplate branch-line 3dB coupler based on the planar circuit theory, which takes into account the electromagnetic field disturbance at transmission line junctions and its mutual interaction. Moreover, the corresponding branch-line 3dB coupler analyzed and synthesized here is constructed practically and measured for 1-12GHz. Since the synthesized results agree well with the measured results, the validity and the usefulness of this planar circuit theory is confirmed.

## 1. Introduction

A branch-line 3dB coupler shown in Fig.1 is an important circuit in the microwave integrated circuits. The operating mechanism is based on transmission line theory which is already well known. In these high-frequency bands where the strip line width is not negligible compared with the wavelength, the presence of the electromagnetic field disturbance at transmission-line junctions and its mutual interaction has strong effects on frequency characteristics. Hence, a systematic method of circuit analysis and synthesis is strongly needed, which takes into consideration those effects [1,2].

The content of this paper is as follows.

In Section 2, we present how to model triplate branch-line 3dB coupler by planar type transmission-line and how to analyze these circuit systematically by the planar circuit theory. In Section 3 and 4, we present how to synthesize the triplate branch-line 3dB coupler based on working mechanism in each section of the coupler, i.e., branch-lines and junctions.

## 2. Analysis of Branch-Line 3dB Coupler based on Planar Circuit Approach

The triplate branch-line 3dB coupler can be approximated by the planar circuit model whose boundary is surrounded by a magnetic wall as shown in Fig.1. Then its internal electromagnetic field is given by  $E = (0, 0, E_z)$  and  $H = (H_x, H_y, 0)$ .

Consequently, voltage and current densities in the planar circuit are defined as

$$V(x, y) = -E_z \cdot d \quad J(x, y) = (H_y, -H_x) \quad (1)$$

The relations of voltage and current densities are given by the planar circuit equations (2) and (3)

$$\text{grad } V(x, y) = -j\omega\mu d \cdot J \quad (2)$$

$$\text{div } J(x, y) = -j\omega\epsilon/d \cdot V \quad (3)$$

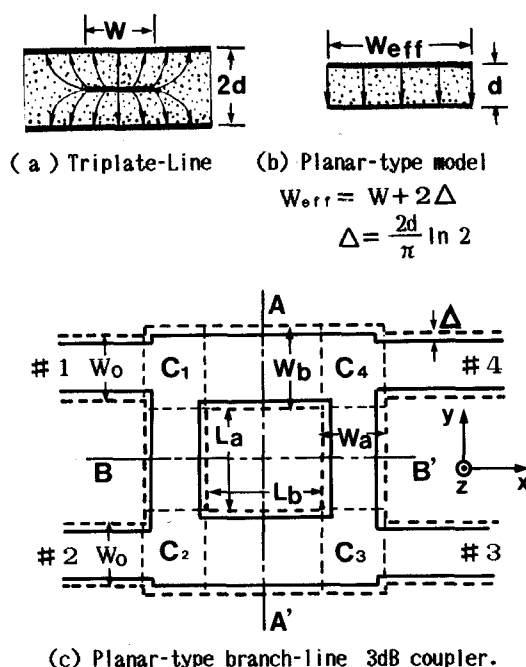


Fig.1 Planar-type transmission-line branch-line 3dB coupler.

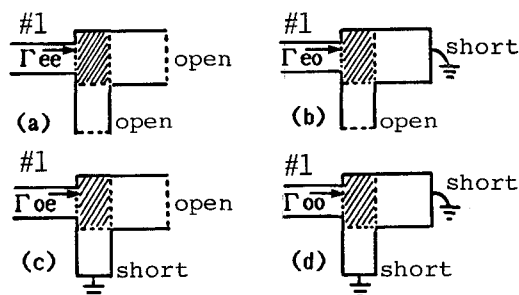


Fig.2 1/4-branch-line 3dB coupler and terminated conditions.

The frequency characteristics of a planar-type coupler are obtained in principle by solving the planar circuit equations (2) and (3) under the given boundary and excitation conditions [3].

Since the planar type 3dB branch-line coupler has a double symmetry with respect to A-A', B-B' as shown in Fig.1, scattering-matrix then are given by eq.(4) [4].

$$\begin{aligned} S_{11} &= (\Gamma_{ee} + \Gamma_{oe} + \Gamma_{eo} + \Gamma_{oo}) / 4 \\ S_{21} &= (\Gamma_{ee} - \Gamma_{oe} + \Gamma_{eo} - \Gamma_{oo}) / 4 \\ S_{31} &= (\Gamma_{ee} - \Gamma_{oe} - \Gamma_{eo} + \Gamma_{oo}) / 4 \\ S_{41} &= (\Gamma_{ee} + \Gamma_{oe} - \Gamma_{eo} - \Gamma_{oo}) / 4 \end{aligned} \quad (4)$$

where these complex reflection coefficients  $\Gamma$  can be calculated by the effective impedance of a dominant TEM-mode looking at the reference terminal port of the 1/4 circuit shown in Fig.2. This effective impedance can be calculated by the planar circuit theory (normal mode expansion), where a contributions of the electromagnetic field disturbance at the junctions has been taken into account [5].

Based on above theory, the frequency characteristics of basic 3dB coupler circuits shown in Fig.3 are calculated for several operating center frequency ranged from 1 to 6 GHz where input/output transmission line is connected to the coupler in the right angle. Some typical results are shown in Fig.3 as function of frequency. Here, the enough number of normal mode are taken into account so as to satisfy the required accuracy for dc to 12 GHz. From the calculated results, following items can be pointed out:

(1) Up to around 1 GHz, the effect of the junction reactance caused by disturbance on frequency characteristics is not serious.

(2) With increasing center frequency, although the optimization of connecting positions of the input/output line and branch-line length are carried out, final calculated results show only a little improvement of the characteristics and the desired characteristics can not be recovered. For a further improvement of characteristics, it is found that the optimum design of planar-type three port junction circuit is needed.

### 3. Synthesis Procedure

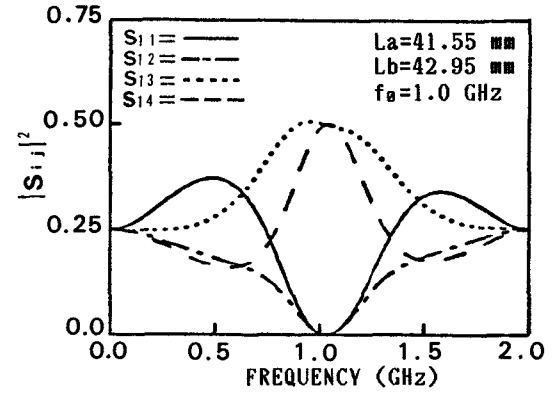
In order to realize the desired frequency characteristics, it was pointed out that a planar-type three port junction circuit must be designed properly, considering the disturbance of the electromagnetic field at the junction.

Generally, transmission-line branch-line 3dB coupler consists of the following two functions.

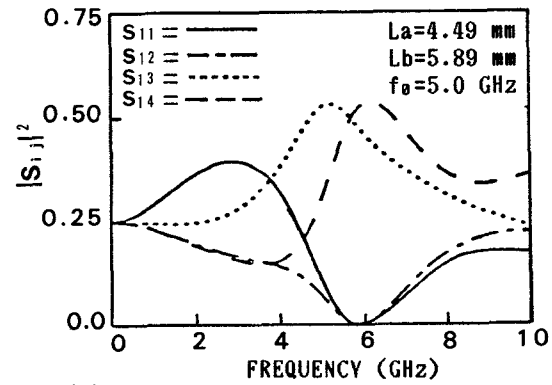
(1) A function of three port junctions:

S-matrix  $[S_0]$  of an idealized three port junction circuit having characteristic impedance  $Z_0, Z_0$  and  $Z_0/\sqrt{2}$  shown in Fig.4(a) is given by [5]

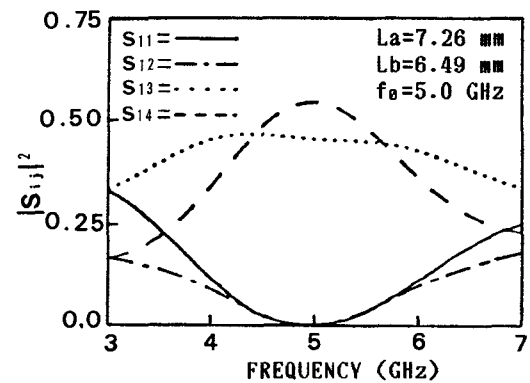
$$[S_0] = \frac{1}{\sqrt{2} + 1} \begin{bmatrix} -1 & \sqrt{2} & \sqrt{2\sqrt{2}} \\ \sqrt{2} & -1 & \sqrt{2\sqrt{2}} \\ \sqrt{2\sqrt{2}} & \sqrt{2\sqrt{2}} & -(\sqrt{2} - 1) \end{bmatrix} \quad (5)$$



(a) center frequency  $f_0 = 1$  GHz



(b) center frequency  $f_0 = 5$  GHz



(c) characteristics after compensations of branch-line length  $f_0 = 5$  GHz

Fig.3 Frequency characteristics of basic branch-line 3dB coupler designed on the transmission-line theory.

(2) A function of branch-line( $L=\lambda g/4$ ):

Branch-line of line length  $L=\lambda g/4$  has the phase shifts of  $90^\circ$ .

Thus, the equivalent circuit of transmission line 3dB coupler is given in Fig.5 from the functional viewpoint. In order to synthesize the planar-type branch-line 3dB coupler having the same frequency characteristics with transmission-line branch-line coupler, we should synthesize a planar type three port junction and branch-line which satisfy the above mentioned two functions.

The synthesis procedure can be summarized as a following steps:

Step 1. A design of planar-type branch-line width  $W_a$  and  $W_b$  ( $W_a=W_0, W_b=\sqrt{2} W_0$ ).

$W_a$  and  $W_b$  can be calculated from the characteristic impedance  $Z_0$  and  $Z_0/\sqrt{2}$ .

Step 2. A design of planar-type three port junction circuit:

We have to design the planar-type three port junction which has idealized three port junction character given by eq.(5) at the center frequency. S-parameters of practical planar-type three port junction circuit becomes complex number.

However, if the pattern of planar-type three port junction circuit shown in Fig.4(b) is synthesized to satisfy eq.(6),

$$|S_{11}| = |S_{01}| \quad (6)$$

then, S-matrix of the planar-type three port junction circuit can be expressed as follows:

$$S = P \cdot S_0 \cdot P \quad (7)$$

$$P = \text{diag.}(e^{-j\theta_1}, e^{-j\theta_2}, e^{-j\theta_3})$$

In equivalent circuit in Fig.5, it is possible to compensate the electric angle by line length of branch-line, which is explained in the next step.

Step 3. A design of branch-line length  $L_a$  and  $L_b$ :

The line-length  $L_i$  is redesigned to compensate the phase shift  $\theta_i$  of planar-type three port junction circuit. That is,

$$L_i = (1 - 2\theta_i/90) \cdot \lambda_g/4 \quad (8)$$

#### 4. Results of Synthesis and Experiment

In order to synthesize a planar-type 3dB branch-line coupler, the frequency characteristics of the three port junction circuit shown in Fig.6 are calculated from dc to 15 GHz with connecting angle  $\theta$  as a parameter. Based on the calculated results shown in Fig.6, the connecting angle of  $\theta = 31^\circ$  is taken, where eq.(6) holds approximately. The total frequency characteristics of planar-type three port junction circuit ( $\theta = 31^\circ$ ) is shown in Fig.7.

The planar-type branch-line coupler are synthesized, following the procedure of section 3. The frequency characteristics of the final planar-type 3dB coupler are calculated and shown in Fig.8. The corresponding circuits is fabricated and experiment results is given in Fig.8.

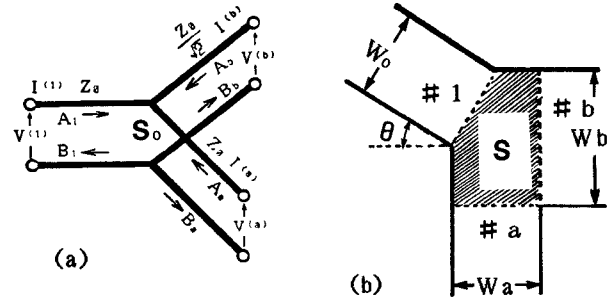


Fig.4 Three-port junction circuit.

(a) transmission-line junction( $S_0$ )

(b) planar-type junction( $S$ )

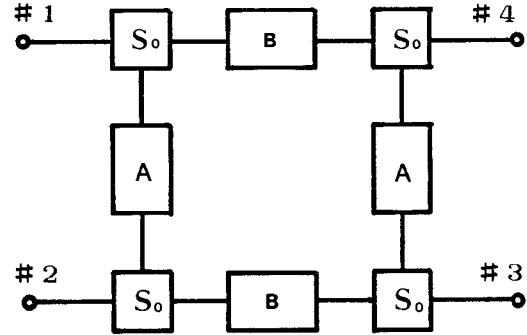


Fig.5 Function of each section of branch-line 3dB coupler after compensations of line-length.

S: planar-type junction

A: line-width  $W_a$ , line-length  $L_a$

B: line-width  $W_b$ , line-length  $L_b$

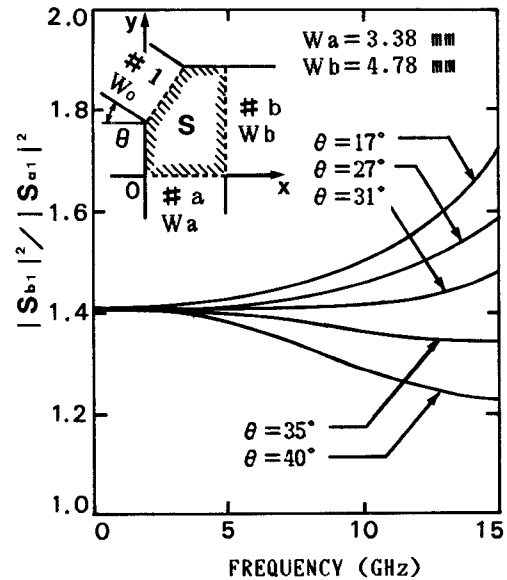


Fig.6 Frequency characteristics of three-port junction with coupling angle of port 1.

## 5. Conclusion

In order to establish a new analysis and synthesis method of a strip-line branch-line 3dB coupler, which takes into account the electromagnetic field disturbance at the junction discontinuity, the triplate branch-line 3dB coupler is modeled by the planar circuit with a magnetic wall.

We have described a systematic method of analysis and synthesis of the planar-type branch-line 3dB coupler based on the planar circuit theory. Moreover, the method of analysis and synthesis mentioned in this paper is applied to practical cases. Since the calculated results based on this method of analysis and synthesis agree well with the measured results, the validity and the usefulness of this method is confirmed.

## Acknowledgment

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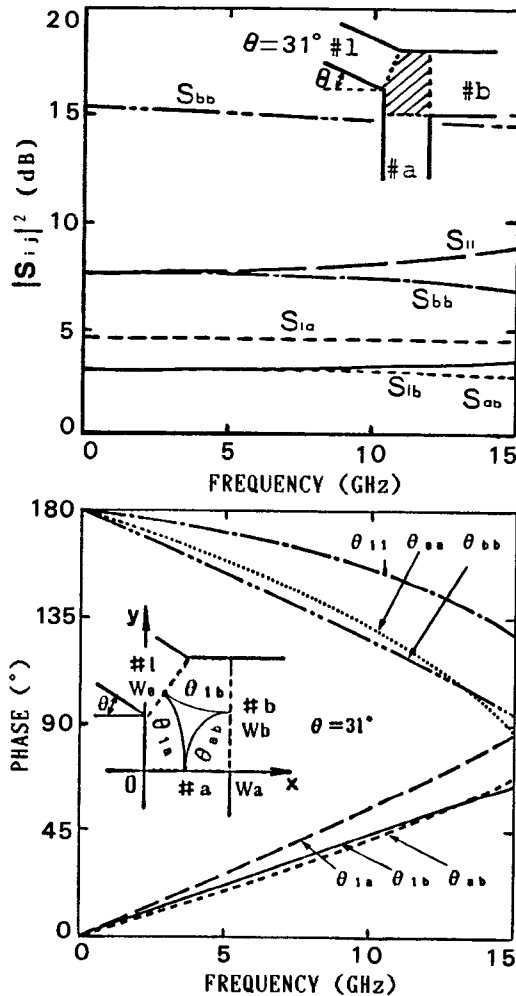
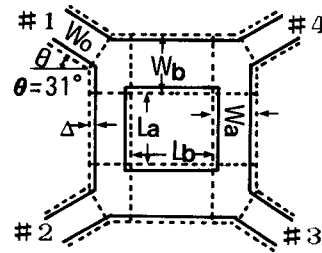


Fig.7 Frequency characteristics of planar-type three port junction.

## References

- (1) T. Okoshi, "Planar Circuits for microwaves and lightwaves", Springer-Verlag, 1985.
- (2) Hsu, Jui-Pang and T. Anada, "Planar circuit equation and its practical application to planar-type transmission-line circuit", in IEEE MTT Int. Symp. Dig. pp. 574-676, 1983, etc.
- (3) Hsu, Jui-Pang, T. Anada and H. Kaneko, "Computer analysis of microwave planar circuit with impedance matrix", Trans. I.E.C.E., Japan J64-B, 9, pp. 986-993 (Sept. 1981) (in Japanese).
- (4) C. G. Montgomery, R. H. Dicke and E. M. Purcell "Principles of Microwave Circuits", New York McGraw-hill, 1948.
- (5) T. Anada and Hsu, Jui-Pang, "A Synthesis of Strip-Line Type 3dB Hybrid Circuit based on Planar Circuit Equations", Paper of Technical Group, TGMW84-85 (Dec. 1984) (in Japanese)



planar-type transmission-line				practical triplate-line			
Wa	Wb	La	Lb	Wa'	Wb'	La'	Lb'
3.38	4.78	6.81	7.68	2.10	3.50	5.53	6.40

dimension : mm  $\Delta = 0.64$  mm  
Rexolite 2200 ( $\epsilon_s = 2.62, d = 1.45$  mm)

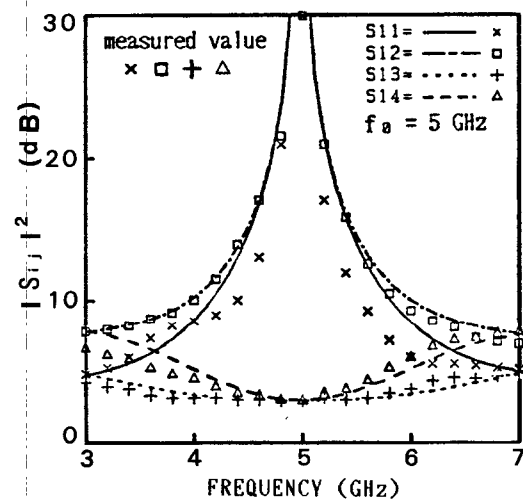


Fig.8 Calculated and measured frequency characteristics of branch-line 3dB coupler designed on this synthesis method.